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$$\frac{(x-ae)^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1\ldots(1);\quad (x-ae)^2+y^2=a^3\ldots(2);\quad y=mx\ldots(3).$$

Solving (1) and (3), we find $x=\frac{a(1-e^2)[e\pm\sqrt{(1+m^2)}]}{1-e^2+m^2}=k$, say. Then $y=mk$. Hence, the coördinates of the center of the circle described on the focal radius are $\frac{1}{2}k$, $\frac{1}{2}mk$, and the equation of this circle is

$$(x-\tfrac{1}{2}k)^2+(y-\tfrac{1}{2}mk)^2=\tfrac{1}{4}k^2(1+m^2)\ldots(4).$$

Solving (2) and (4), we find that the discriminant of the resulting quadratic equations vanish, showing that the roots are equal. Hence, (4) intersects (2) in two coincident points and is, therefore, tangent to it.

CALCULUS.

222 (Incorrectly numbered 221, p. 117). Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, O.

If a, b, c, ... represent all the prime numbers 2, 3, 5, ... prove that

$$\left(1+\frac{1}{a^2}\right)\left(1+\frac{1}{b^2}\right)\left(1+\frac{1}{c^2}\right)\ldots\ldots=\frac{15}{\pi^2}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{5^2}\right)\left(1+\frac{1}{7^2}\right)\cdots\Big)$$

$$=\frac{\left(1-\frac{1}{2^4}\right)\left(1-\frac{1}{3^4}\right)\left(1-\frac{1}{5^4}\right)\left(1-\frac{1}{7^4}\right)\cdots}{\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{7^2}\right)\cdots}=\frac{\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots}{\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\cdots}$$

$$=\frac{\pi^5/6}{\pi^4/90}=\frac{15}{\pi^2}.$$

(See Vol. V, No. 5, May, 1898, page 134, third line from bottom, together with what precedes, for the complete solution. Also see Vol. XIII, No. 2, February, 1906, pages 40—41.)

223. Proposed by O. E. GLENN, Ph. D., Philadelphia, Pa.

Prove that $\lim\limits_{n\doteq\infty}\frac{\sum\limits_{\lambda=1}^{n}\lambda^k}{n^{k+1}}=\frac{1}{k+1}$.

I. Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

Putting $h=\frac{1}{n}$, we get $\lim\limits_{\substack{h\doteq 0\\ nh=0}} h[h^k+(2h)^k+(3h)^k+\ldots+(nh)^k]$

$$=\int_0^1 x^k dx=\frac{1}{k+1}.$$

II. Solution by EDWIN L. RICH, Schenectady, N. Y.

Here $\sum\limits_{\lambda=1}^{n}\lambda^k=\frac{n^{k+1}-1}{k+1}$, treating the summation as an integration. Then we have $\mathop{\text{Limit}}\limits_{n\doteq\infty}\left(\frac{1}{(k+1)}\right)\left(\frac{n^{k+1}-1}{n^{k+1}}\right)$. Since this is in the form $\frac{\infty}{\infty}$, differentiate the numerator and denominator $k+1$ times, dividing numerator and denominator by $(k+1)$, (k), $(k-1)$, ..., $(k-k+1)$ successively, and we have

$$\lim_{n\doteq\infty}\frac{\sum\limits_{\lambda=1}^{n}\lambda^k}{n^{k+1}}=\frac{1}{k+1}.$$

We might arrive at the same conclusion, by considering n finite, then $\frac{1}{k+1}\left(\frac{n^{k+1}-1}{n^{k+1}}\right)=\frac{1}{k+1}\left(1-\frac{1}{n^{k+1}}\right)$, which when $n\doteq\infty$, approaches $\frac{1}{k+1}$ as a limit.

III. Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

One of the most general formulae for the summation of series is that of Euler, who has stamped his genius upon so many branches of mathematics. It is

$$\Sigma u=\int u\partial x+\tfrac{1}{2}u+\frac{B_1}{1.2}.\frac{\partial u}{\partial x}-\frac{B_3}{4!}\frac{\partial^3 u}{\partial x^3}+\frac{B_5}{6!}.\frac{\partial^5 u}{\partial x^5}-\ldots+\ldots$$

where B_1, B_3, B_5, denote the famous Bernoulli's Numbers $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{42}$, etc. Putting $u=x^k$, we have

$$\Sigma x^k=\frac{x^{k+1}}{k+1}+\tfrac{1}{2}x^k+\frac{B_1}{2}.\left(\frac{n}{1}\right)x^{k-1}-\ldots$$

Consequently we have

$$\frac{\Sigma n^k}{n^{k+1}}=\frac{n^{k+1}}{n^{k+1}}\left(\frac{1}{k+1}+\tfrac{1}{2}.\frac{1}{n}+\tfrac{1}{6}\cdot\tfrac{1}{2}.\frac{1}{n^2}+\ldots\right)=\frac{1}{k+1}+\tfrac{1}{2}.\frac{1}{n}+\ldots$$

For $n=\infty$, all the terms save the first cancel, and we have $\frac{1}{k+1}$ as the limiting value. The sum of the kth powers of the natural series of numbers can also be found without the aid of calculus by the theory of undetermined coefficients.